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DR 5.5: Combining basic cross-modal concepts into novel concepts

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Cross-modal learning is an important characteristic of a system that is supposed to be capable of self-extension. The system should exploit different modalities and extend its current knowledge based on the information obtained from different sources and based on the previously learned concept models. In this deliverable we address the cross-modal learning in different domains, ranging from self-supervised learning of object affordances through hierarchical learning of representation of space to combining perception from different modalities to facilitate high-level cross-modal learning.

Executive Summary

An important characteristic of a robot that operates in a real-life environment is the ability to continuously expand its current knowledge, in a lifelong manner. The system has to create concepts by observing the environment and also to extend these concepts and create novel concepts on top of them while interacting with the environment as well as with other cognitive agents and humans. Interactive continuous cross-modal learning, which is the main research topic of Workpackage 5, is therefore an essential characteristic of a self-extending cognitive system.

Different types of cross-modality are addressed in this deliverable. Firstly, we present cross-modal learning of object affordances and action effects; here the information arising from the visual subsystem is combined with the information from the manipulation subsystem. Also, different derived modalities, or cues, from the visual subsystem are taken into account: colour, depth (which is converted in 3D point cloud), and motion.

Then we present a hierarchical approach to building the representation of space. Range data captured by the robot is used to learn a hierarchy of so called parts. The parts represent concepts about spatial shape primitives, which are very simple on the lowest layer, and are then combined into more complex parts in the upper layers of the hierarchy.

And finally, we also address the problem of binding of modal concepts from different modalities that facilitates high-level cross-modal learning.

Some of the work presented in this deliverable is a continuation of the work performed in the previous years and mostly presented in the deliverables DR.5.1. to DR.5.4. The work about hierarchical learning of space is also highly related to Workpackage 3 on spatial cognition and the work on learning action effects is very related to Workpackage 2 and the deliverable DR.2.5 on models of object behaviour. There is also an overlap between the work on cross-modal binding and learning presented in this deliverable and the Workpackage 1 deliverable DR.1.5 on representations of gaps in knowledge, since it is about beliefs, which play an important role in both processes, cross-modal information fusion and learning, and knowledge gap representation and management. This work is also highly related to Workpackage 7, since it presents the main principle for binding and reference resolution implemented in the George system.

Role of Combining basic cross-modal concepts into novel concepts in CogX

In the process of continuous interactive cross-modal learning, the system tries to understand what it does know and what it does not, and act accordingly with the goal of updating the current concepts and building novel concepts on top of them. Therefore, the main research topic fits very well with the main motto of the project: to self-understand to be able to self-extend.

Contribution to the CogX scenarios and prototypes

In order to monitor and show progress on active and interactive continuous learning, we have designed the George scenario (Interactive cross-modal learning scenario) [37] (see also deliverable DR.7.5). This scenario has been designed as a use case for guiding and testing the system-wide research and for demonstrating methods developed in WP 5 and in some other workpackages in a complex integrated working system. The management of beliefs and cross-modal binding presented in this deliverable form the central part of the George system, crucial for a consistent fusion of information from different subsystems and for enabling consistent behaviour of the very heterogeneous system.

1 Tasks, objectives, results

1.1 Planned work

This deliverable mainly tackles the problems addressed in Task 5.4 of Workpackage 5:

> *Task 5.4: Combining concepts into novel concepts. Develop a system that is able to combine concepts learned in the previous tasks into novel concepts; to learn complex concepts and hierarchies of concepts.*

As such, it is addressing the following objectives as specified in the Technical annex:

1. *A unified framework for representing beliefs about representations of action effects, observation models, incomplete information and categorical knowledge. [WPs 1,4,5]*

5. *A theory of how to use these representations to identify learning opportunities, plan and execute plans in order to learn so as to perform future tasks more effectively and efficiently. [WPs 4,5]*

8. *New representations and algorithms to allow a robot to extend its categorical knowledge by identifying gaps and learning the relationships between different modalities (e.g. vision and language). [WP 5]*

We will structure this deliverable according to several research lines that have been addressed. First, let us look at our plans and goals that we had set:

- **Self-supervised learning of object affordances.**

  We planned to develop an algorithm for inducing causal relationships of action/object complexes, in terms of the trajectory of objects, represented as a sequence of object poses, given some motor action (here a pushing action). For this purpose, a quantization algorithm needed to be developed which can discretize the sensorimotor space. Additionally, an algorithm that can extract causal relationships in form of probabilistic transitions among discrete states had to be devised, based on an algorithm for extracting substochastic sequential machines (CrySSMEx) [15] from dynamic systems.

  The CrySSMEx algorithm can find qualitative states depending on the output function given by the output space. Thus, we planned to cluster states that represent more abstract concepts from the sensorimotor space of a pushing scenario.

In addition to this, we also planned on improving our previously proposed algorithm for self-supervised cross-modal learning [31] by using additional mechanisms to enhance its performance at acquiring novel affordance concepts over short-term training periods. Specifically, we were interested in developing feature relevance determination algorithms that could rapidly find the most discriminative feature dimensions in the input space for predicting the naturally occurring categories in the output space.

- **Learning hierarchical representation of space.**

  The goal for this year was to develop an algorithm for learning a compositional hierarchical representation of space based on data obtained with a range sensor. We aimed to extend the existing Learning the Hierarchy of Parts algorithm [8], by incorporating rotational invariance of parts into the model. We planned to learn as many layers of the hierarchy as possible using only range scans as input data. We sought to evaluate the performance of the learned concepts through the room classification problem, and to validate our model in comparison with other, state-of-the-art, approaches.

- **Cross-modal binding and learning.**

  In Year 3 the cross-modal learning and binding concepts were used in parallel to the belief structures. A Markov Logic Network engine component was used for reference resolution. The aim for this year was to enhance the dialogue between the robot and the tutor, which in turn requires new enhanced reference resolution MLN. We also wanted to bring MLN reasoning into the belief structure itself. MLNs should have an important role in propagation of information between various types of beliefs.

1.2 Actual work performed

1.2.1 Self-supervised learning of object affordances

We developed an online learning algorithm for quantization of spaces in the pushing affordances scenario. After the robot performs a pushing action, a density estimation (quantization) algorithm runs for the current sequence of effector and object poses obtained, in order to estimate the density of this sensorimotor space. Additionally, an output space is also discretized, which corresponds to changes in rotation of the object. The output space is needed to split the state space, which are representations of object poses at the next time step. The quantization algorithm is based on the incremental Growing Neural Gas (GNG) algorithm and the Minimum Description Length principle for evaluation of clustering performance [33]. When a GNG network or graph reaches a stable state, the algorithm stops and a new action is performed by the robot to continue learning.

After the quantization is performed, a state representing a set of instances is split when the output or the next state differs (so that the entropy is high). The transitions among states and its probabilities are obtained to construct probabilistic machines that represent the behavior of these action/object dynamical systems.

By using the extracted substochastic sequential machines (SSMs), we were able to predict the object behavior accurately.

When the sensorimotor space is huge, it is appropriate to split it in different regions where different learning machines (in this case quantizers) can be employed. We used these divide-and-conquer approaches to accelerate the learning process and make it more efficient. The regions are split after some time step and by employing a measure of variance in the sensorimotor data sets.

The CrySSMEx algorithm also splits the state space in a similar way but using other criteria, as explained above. However, it is in principle a similar process. At the end, the state space is represented by a tree of Vector quantizers, resembling a hierarchical clustering.

Once we obtained a quantization of the input space, we use these quantizers with other representations for the output space. We then discretized the output space in such a way that we can distinguish among abstract object behaviors like sliding, flipping over and tilting. The state space quantizer then groups the states in a different way, splitting the space according to this new output function. Thus, these new state space regions might be viewed as components of a joint distribution.

Once again by using these new SSMs, we can predict with high accuracy the classifications of object behavior (see Annex 2.1).

In our other work on self-supervised cross-modal learning described in last year's deliverable [31], we have developed methods for feature relevance determination [32] that serve to augment the original algorithm. These methods stem from ideas originally touched upon in the previous year's work, but have been more thoroughly developed, investigated and evaluated this year. They are based on the idea of applying the Fisher criterion score to learning vector quantization algorithms for online feature relevance determination (see Annex 2.2).

In the attached paper [32], two new algorithms for LVQ-based relevance determination are presented. Both methods exploit the positioning of the prototype vectors in the input feature space to inform estimates of the Fisher criterion score along the input dimensions, which are then used to form online estimates of the relevance of the input dimensions with respect to the classifier output. Both methods provide online updates that may be used alongside regular LVQ updates or within the broader context of our self-supervised cross-modal learning framework and neither method requires the specification of a learning rate, as in stochastic gradient descent. Performance advantages are demonstrated in experiments on various popular classification datasets, as well as on data from our object push affordance learning experiments.

1.2.2 Learning hierarchical representation of space

In work described in Annex 2.3 we propose a new compositional hierarchical representation of space, which is learned based on statistically significant observations. We have focused on a two dimensional space, since many robots perceive their surroundings in two dimensions using range sensors. Range data is transformed into images and then a hierarchy of so called parts is learned from those images. Parts, which are rotationally invariant, represent concepts about spatial shape primitives. They are very simple on the lowest layer, while their complexity and size increases with respect to the height of the corresponding layer of the hierarchy. At the bottom, concepts are represented as small fragments of lines in several different orientations. On higher layers compositions of lower layer concepts are learned, forming more and more complex shapes. Only shapes that have been observed most frequently in the images used for learning are memorized, and then used to model the environment. Only a few lower layers of the proposed hierarchy are currently being learned. In the future, the image formation step will be omitted and information from other modalities, like odometry, will be used to combine the information from separate range scans into a unified map, which will provide a more complete view of the environment. Based on these maps even more complex shapes will be formed, which will introduce the abstraction needed to learn higher level concepts. These will provide good scalability of the model through sharing of same concepts between different room categories. A cognitive system using our representation of space would be able to make use of a large quantity of information, that has been obtained in past observations, to extend it's knowledge about general characteristics of space, and then use this knowledge as a compact and expressive description of it's surroundings.

In this work we also propose a new low-level image descriptor, by which we demonstrate the performance of our representation in the context of the room classification problem based only on data obtained with a laser range finder. Using only the lower layers of the hierarchy, we obtain state-of-the-art classification results on demanding datasets. Room classification methods, which are based on data obtained with range sensors have a potential to work faster than other, for example, vision based, approaches, since the input information is of lower dimensionality, while on the other hand, the stinginess of the data makes this approach much more demanding. Such approach could therefore provide a cognitive system with a quick first impression about room type, which could then serve as a reliable basis for the use of temporally more costly classifiers to verify the proposed hypothesis.

1.2.3 Cross-modal binding and learning

In work described in Annex 2.4 we devised a new belief scheme that now also supports MLN reasoning. The beliefs form a cognitive layer where multi-modal and multi-agent information is associated and merged to a-modal representations. In general a belief can be regarded as a high-level representation of an element of the physical reality, grounded in one or more sensory inputs, attributed to a specific agent or a combination of both. The new belief scheme distinguishes five distinct belief categories. *Private* beliefs reflect the robot perceptions of the environment based on its sensory input. *Assumed* beliefs are used to establish cross-agent or cross-modal common ground; they are created from private beliefs by translating the modal symbols to the a-modal ones. *Attributed* beliefs contain information that a robot attributes to another agent. *Verified* beliefs are created from attributed beliefs; they essentially contain the acknowledged information from the attributed beliefs. *Merged* beliefs combine information from verified and assumed beliefs and represent the final a-modal situated knowledge, ready to be used by the higher level cognitive processes (e.g. motivation, planning). They contain as reliable information as possible and as much information as available.

MLN components have a triple role in this Belief scheme: (i) They are used for binding — the binding process associates between beliefs from different modalities or different epistemic origins (in George the binding principles are used in reference resolution), (ii) as a translator between modal and a-modal symbols and (iii) for information fusion. In the information flow from sensory data to higher cognition, the information fusion can be regarded as a next step after the binding.

1.3 Relation to the state-of-the-art

In this section we discuss how our work is related to, and goes beyond the current state-of-the-art.

1.3.1 Self-supervised learning of object affordances

The problem of object prediction has already been tackled by using offline learning algorithms [20], which lack incremental ways of learning when new data sequences are added. In this new approach, we can estimate the density of sensorimotor spaces in an online way.

We also extract probabilistic machines that can be used in planning tasks a posteriori, since they are essentially graphs on which some reasoning methods could be applied. They also encode an entropy based representation of causal relationships that can be used for active learning.

We obtain qualitative representations of temporal sequences of action/object complexes, taking into account trajectory information like object and robot poses. Traditionally, learning algorithms with abilities of temporal processing like Hidden Markov Models (HMMs) [10], Recurrent Neural Networks (RNNs) [34] and Dynamic Bayesian Networks [26] have been used in robotic learning tasks. Substochastic sequential machines have been extracted from RNNs to extract qualitative information learned by the RNNs [15, 11]. Substochastic Sequential Machines are similar to HMMs, in that they are probabilistic finite state representations. Some characteristics of SSMs like entropy based representation of uncertainty might be advantageous when designing information-theoretic active learning methods.

Learning vector quantization (LVQ) [18] provides an intuitive, and often highly effective, means for discriminative learning where prototype vectors are used to quantize the input feature space and given labels to form piecewise-linear classifiers using the nearest neighbour rule. Since their introduction, LVQ algorithms have undergone various analyses and seen various improvements to their design and much attention has also been paid in recent years to the role that the distance metric plays in the effectiveness of LVQ methods, which was the focus of our investigation in [32]. LVQ ordinarily relies on the Euclidean metric to measure the distance between data points, which provides equal weighting to all input dimensions. Many of the input dimensions, however, may have little relevance when considering the desired output function and may even have a detrimental effect on the output if considered with equal weighting in the metric to the more important dimensions. One standard approach to this issue is to pre-process the data using some form of feature selection or dimensionality reduction, but this can be infeasible in many learning scenarios where the training data are not available in advance, e.g. autonomous robotics.

One early adaptation of LVQ3 known as *distinction sensitive learning vector quantization (DSLVQ)* [28] achieves this by using a heuristic to adjust weights along each of the input dimensions to modify the Euclidean metric. An adaptation of LVQ1 known as *relevance learning vector quantization (RLVQ)* [3] uses Hebbian learning to do similar, by adjusting weights for each of the input dimensions at every training step depending on whether they contributed to the correct or incorrect classification of a training sample. RLVQ was subsequently adapted for use with GLVQ producing a method known as *generalized relevance learning vector quantization (GRLVQ)* [13] such that the dimensional weight updates also adhere to gradient descent dynamics in a similar way to the prototype updates. Another modified version of GLVQ [43] uses Fisher's discriminant analysis to create an alternative metric to the weighted Euclidean distance that employs a matrix transformation to reduce the feature space dimensionality. More recently, an adaptive metric was used in combination with training data selection for LVQ [27].

By comparison, in our work described in Annex 2.2, an advantage provided by the proposed methods over other metric-adaptive LVQ methods based on gradient descent, is that they do not require a learning rate or other parameters to be specified. Moreover, they provide incremental update rules that operate alongside regular LVQ update rules and can therefore be applied to any algorithms based on the general LVQ paradigm. Experimental evaluations were provided under various stress conditions and over various datasets and the proposed methods were shown to perform competitively against various other LVQ-based methods, and against SVM.

1.3.2 Learning hierarchical representation of space

Numerous spatial models have already been proposed. Metric representations use sensory information to accurately describe the geometry of space to some desired extent [6, 1], topological representations use graphs to model space [40, 5], hybrid approaches combine both of the above paradigms [41, 42], while combining of one or even both of the approaches, metric and topological, on multiple levels of abstraction results in hierarchical representations [30, 21, 25, 44]. Perhaps the closest to our work is the work presented by Mozos [25]. He generates a topology of the environment for room classification based on laser scans. A major difference between his and our idea is that he uses occupancy grids under the topological level, which are not suitable for modelling large environments, since they scale poorly. In our approach, rooms will be represented with parts, which will be shared between different categories, and thus requiring less memory. Despite several existing approaches, to the best of our knowledge, our work is the first attempt of using a hierarchical compositional model for the representation of space on the lowest semantic level, at which range sensors are usually used to observe the environment.

However, compositional hierarchies have been used for some time by the computer vision community [8, 9, 19, 7]. In this work we adapt the hierarchical model from [9] to develop a description suitable for representation of space. It turns out that rotational invarance of parts is crucial for obtaining a compact and expressive hierarchy for spatial representation. This property is not present in the model of [9], therefore, we extended the model to satisfy the above condition.

Various systems performing topological localization have been developed for room classification. In [30] very accurate room classification is achieved using multimodal information. Approaches using less information available for classification have also been considered. Laser range data combined with vision was used for classification in [24], and many approaches that use vision only for the accomplishment of this task have also been presented [29, 46, 2, 48]. The most related to our work are the approaches performing room classification based only on data obtained with range sensors. In [39]

3D Time-of-Flight infrared sensor was used for acquiring 3D information, which allowed the distinction between three types of rooms (office, meeting room and hall). Only laser range data was used in [23]. Their robot was equipped with a 360 degree field of view range sensor and they were able to distinguish between four classes (rooms, corridors, doorways and hallways). The classification was performed with AdaBoost and it was based only on a single scan. Laser range data was also used for classification in [12], where Voronoi random fields (VRFs) were employed to label different places in the environment, providing the distinction between four classes (rooms, hallways, junctions and doorways). Their approach uses a state-of-the-art SLAM technique to generate a metric occupancy grid map of an environment, while the Voronoi graph is then extracted of this map. For each point on the Voronoi graph, VRFs then estimate the type of place it belongs to. Our approach to room classification is based on the proposed hierarchical model. We have taken into consideration a set of room types (living room, corridor, bathroom, and bedroom), which are in our opinion more demanding than the ones presented in the related work [23, 12].

1.3.3 Cross-modal binding and learning

Many of the past attempts at binding information within cognitive systems were restricted to associating linguistic information to lower level perceptual information. Roy et al. tried to ground the linguistic descriptions of objects and actions in visual and sound perceptions and to generate descriptions of previously unseen scenes based on the previously accumulated knowledge [35, 36]. This is essentially a *symbol grounding problem* first defined by Harnad [14]. Chella et al. proposed a three-layered cognitive architecture around the visual system with the middle, *conceptual layer* bridging the gap between linguistic and sub-symbolic (visual) layers [4]. Related problems were also often addressed by Steels [38].

Jacobsson et al. approached the binding problem in a more general way [17] [16] developing a cross-modal binding system that could form associations between multiple modalities and could be part of a wider cognitive architecture. The cross-modal knowledge was represented as a set of binary functions comparing binding attributes in pair-wise fashion. A cognitive architecture using this system for linguistic reference resolution was presented in [45]. This system was capable of learning visual concepts in interaction with a human tutor. A probabilistic binding system was developed within the same group that encodes cross-modal knowledge into a Bayesian graphical model [47]. In [22] a framework for constructing high-level cognitive representations of the environment, called beliefs, was presented. Markov logic was used as the main framework for various types of inference over beliefs, including perceptual grouping, which comes very close to our definition of binding. All these systems ([17] – [22]) assumed static cross-modal knowledge.

2 Annexes

2.1 Roa et al. "Online Density Estimation in a Robotic Manipulation Scenario and its application to Learning of Temporal Action/Object Models and Concepts"

Bibliography S. Roa and G.-J. Kruijff: "Online Density Estimation in a Robotic Manipulation Scenario and its application to Learning of Temporal Action/Object Models and Concepts". Technical Report, 2012.

Abstract Cognitive Robotics implies the ability of robots to learn from the environment by interacting with it and learning causal relations and associations stemming from these interactions. In this paper, we address the particular problem of interacting by manipulating objects, specifically robot arm pushes. To solve this problem we come up with models which can describe the behaviour of objects given some action. For a learning robot it is essential to learn in an incremental way, after new information is coming, without losing generalization and avoiding overfitting. We tackle this problem firstly by estimating the density of a sensorimotor space after a robot performs a new action by using a modification of the incremental Growing Neural Gas (RobustGNG) algorithm. RobustGNG performs a quantization of the space which is robust to noise and overfitting issues. Subsequently, we infer models useful for prediction of object trajectories in terms of object poses. The same machinery is useful for obtaining more coarse-grained predictions, for instance categorizations of object behaviours. Last, but not least, these prediction models should provide a qualitative temporal description of the state space, so that they can eventually be used in planning tasks. Thus, we infer cause-effect models by using a new version of the CrySSMEx algorithm for extraction of substochastic finite-state machines given the quantization obtained by means of RobustGNG.

Relation to WP This work is directly related to continuous learning of cross-modal concepts, where crossmodality comes from sources like manipulation and vision (in this case simulated). It also explores the problem of deriving categorical knowledge from previously learned tasks, i.e. from density estimation of the sensorimotor space.

2.2 Ridge et al. "Relevance Determination for Learning Vector Quantization using the Fisher Criterion Score"

Bibliography B. Ridge, A. Leonardis, and D. Skočaj: "Relevance Determination for Learning Vector Quantization using the Fisher Criterion Score". 17th Computer Vision Winter Workshop, Mala Nedelja, Slovenia,February 1-3, 2012.

Abstract Two new feature relevance determination algorithms are proposed for learning vector quantization. The algorithms exploit the positioning of the prototype vectors in the input feature space to estimate Fisher criterion scores for the input dimensions during training. These scores are used to form online estimates of weighting factors for an adaptive metric that accounts for dimensional relevance with respect to classifier output. The methods offer theoretical advantages over previously proposed LVQ relevance determination techniques based on gradient descent, as well as performance advantages as demonstrated in experiments on various datasets including a visual dataset from a cognitive robotics object affordance learning experiment.

Relation to WP The two new algorithms were proposed in order to augment the short-term training discriminative capacity of our previously proposed self-supervised cross-modal learning algorithm [31] which is capable of generating novel object affordance concepts autonomously.

2.3 Uršič et al. "Room Classification using a Hierarchical Representation of Space"

Bibliography P. Uršič, M. Kristan, D. Skočaj, A. Leonardis. "Room Classification using a Hierarchical Representation of Space". Submitted to IEEE/RSJ International Conference on Intelligent Robots and Systems IROS 2012, 2012.

Abstract Mobile robots need an effective spatial model for the successful operation in real-world environment. The model should be compact and simultaneously possess large expressive power. Moreover, it should scale well. In this work we propose a new hierarchical representation of space, whose compositional structure is learned based on statistically significant observations. We have focused on a two dimensional space, since many robots perceive their surroundings in two dimensions with the use of a laser range finder or a sonar. We also propose a new low-level image descriptor, by which we demonstrate the performance of our representation in the context of room classification problem. Using only the lower layers of the hierarchy, we obtain state-of-the-art classification results on demanding datasets.

Relation to WP This work proposes a new hierarchical model of space. Spatial shape primitives are being learned by combining simple concepts into more complex ones, forming the hierarchical representation. Based on previous observations, the hierarchy containing most frequently detected shapes is learned and then used to derive new abstract concepts, like room categories. Therefore, the work is related to Task 5.4.

2.4 Vrečko et al. "Associating and merging multi-modal and multi-agent information in a cognitive system"

Bibliography A. Vrečko, A. Leonardis and D. Skočaj: "Associating and merging multi-modal and multi-agent information in a cognitive system". TR-LUVSS-02/2012, University of Ljubljana, Faculty of Computer and information science, May 2012

Abstract A critical ability for every cognitive system operating in a complex environment is the ability to combine several representations of the same physical reality into a single shared representation. Such combined, a-modal representations are then ready to be used by higher level cognitive processes, like motivation and planning. In this work we describe a cognitive layer where multi-modal and multi-agent information is associated and merged to a-modal representations. Furthermore we describe the application of cross-modal binding principles to a specific problem of reference resolution.

Relation to WP The technical report addresses the problem of cross-modal binding and learning, as defined in WP 5. It describes the application of these principles to a concrete problem of reference resolution. Furthermore it describes the belief schema where multi-modal information is associated and merged to a-modal representations.

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Relevance Determination for Learning Vector Quantization using the Fisher Criterion Score *

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Abstract. *Two new feature relevance determination algorithms are proposed for learning vector quantization. The algorithms exploit the positioning of the prototype vectors in the input feature space to estimate Fisher criterion scores for the input dimensions during training. These scores are used to form online estimates of weighting factors for an adaptive metric that accounts for dimensional relevance with respect to classifier output. The methods offer theoretical advantages over previously proposed LVQ relevance determination techniques based on gradient descent, as well as performance advantages as demonstrated in experiments on various datasets including a visual dataset from a cognitive robotics object affordance learning experiment.*

1. Introduction

Learning vector quantization (LVQ) [9] provides an intuitive, and often highly effective, means for discriminative learning where prototype vectors are used to quantize the input feature space and given labels to form piecewise-linear classifiers using the nearest neighbour rule. Since their introduction, LVQ algorithms have undergone various analyses and seen various improvements to their design. The original formulations *(LVQ1, LVQ2, LVQ3)* [9] have been shown to be divergent, inspiring the *generalized learning vector quantization (GLVQ)* algorithm [14] where prototypes are updated such that a stochastic gradient descent is performed over an error function. LVQ algorithms have also been shown to be a family of maximum margin classifiers [3], thus providing excellent generalization for novel data with high-dimensional inputs. More recently, the nearest neighbour rule of LVQ has been modified to a k-nearest neighbours rule using a local subspace classifier [7].

Perhaps just as significantly, much attention has also been paid in recent years to the role that the distance metric plays in the effectiveness of LVQ methods. LVQ ordinarily relies on the Euclidean metric to measure the distance between data points, which provides equal weighting to all input dimensions. Many of the input dimensions, however, may have little relevance when considering the desired output function and may even have a detrimental effect on the output if considered with equal weighting in the metric to the more important dimensions. One standard approach to this issue is to pre-process the data using some form of feature selection or dimensionality reduction, but this can be infeasible in many learning scenarios where the training data are not available in advance, e.g. autonomous robotics. Various reformulations of LVQ have been proposed that can adjust the metric during training such that the impact of the individual input dimensions are dynamically re-weighted during training in accordance with the data under consideration. This can make a crucial difference, both during training for more efficient adjustment of the prototypes, and when classifying test samples where the undue consideration of irrelevant dimensions can mean the difference between a correct and incorrect classification.

One early adaptation of LVQ3 known as *distinction sensitive learning vector quantization (DSLVQ)* [11] achieves this by using a heuristic to adjust weights along each of the input dimensions to modify the Euclidean metric. An adaptation of LVQ1 known as *relevance learning vector quantization (RLVQ)* [1] uses Hebbian learning to do similar, by adjusting weights for each of the input dimensions at every

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training step depending on whether they contributed to the correct or incorrect classification of a training sample. RLVQ was subsequently adapted for use with GLVQ producing a method known as *generalized relevance learning vector quantization (GRLVQ)* [6] such that the dimensional weight updates also adhere to gradient descent dynamics in a similar way to the prototype updates. Another modified version of GLVQ [15] uses Fisher's discriminant analysis to create an alternative metric to the weighted Euclidean distance that employs a matrix transformation to reduce the feature space dimensionality. More recently, an adaptive metric was used in combination with training data selection for LVQ [10].

In this paper, two new algorithms for LVQ-based relevance determination are presented. Both methods exploit the positioning of the prototype vectors in the input feature space to inform estimates of the Fisher criterion score along the input dimensions, which are then used to form online estimates of the relevance of the input dimensions with respect to the classifier output. Both methods provide online updates that may be used alongside regular LVQ updates and neither method requires the specification of a learning rate, as in stochastic gradient descent. The remainder of the paper is organized as follows. In Section 2 the background theory and related algorithms are outlined. The new algorithms are described in Section 3. Experimental results are provided in Section 4 and concluding remarks are provided in Section 5.

2. Related Algorithms

Let $X = \{(\mathbf{x}^i, y^i) \subset \mathbb{R}^n \times \{1, \ldots, C\} \,|\, i = 1, \ldots, N\}$ be a training set of n-dimensional vectors and corresponding class labels. Let $X^c = \left\{(\mathbf{x}^i, y^i) \in X \,\middle|\, y^i = c\right\}$ and $N^c = |X^c|$. Similarly, let $\mathcal{W} = \{(\mathbf{w}^i, c^i) \subset \mathbb{R}^n \times \{1, \ldots, C\} \,|\, i = 1, \ldots, M\}$ be a set of prototype vectors with corresponding class labels, and let $\mathcal{W}^c = \left\{(\mathbf{w}^i, c^i) \in \mathcal{W} \,\middle|\, c^i = c\right\}$ and $M^c = |\mathcal{W}^c|$. Given a vector $\mathbf{x} \in \mathbb{R}^n$, denote its components as $(x_1, \ldots, x_n)$. Letting $\mathbf{x}$ be an n-dimensional data vector and $\mathbf{w}$ be an n-dimensional prototype vector, then a weighted squared Euclidean distance between both vectors may be defined as

$$d^2(\mathbf{x}, \mathbf{w}) = \sum_{i=1}^{n} \lambda_i (x_i - w_i)^2, \qquad (1)$$

where the λ_i are weighting factors for each dimension. Adding such weights to the Euclidean metric allows for the possibility of re-scaling each of the input dimensions depending on their respective influences on the classification output. Moreover, it enables the metric to be made *adaptive* such that the weights are adjusted dynamically during training depending on the data.

Prototype vectors have associated receptive fields based on the metric and classification of samples is performed by determining which receptive fields those samples lie in, or alternatively, which prototype vectors are closest to the samples. The receptive field of prototype $\mathbf{w}^i$ is defined as: $R^i = \{\mathbf{x} \in X \,|\, \forall (\mathbf{w}^j, c^j) \in \mathcal{W},\ d^2(\mathbf{x}, \mathbf{w}^i) \leq d^2(\mathbf{x}, \mathbf{w}^j)\}$. Given a sample $(\mathbf{x}, y) \in X$, we denote by $g(\mathbf{x})$ a function that is negative if $\mathbf{x}$ is classified correctly, i.e. $\mathbf{x} \in R^i$ with $c^i = y$, and is positive if $\mathbf{x}$ is classified incorrectly, i.e. $\mathbf{x} \in R^i$ with $c^i \neq y$. We also let f be some monotonically increasing function.

The goal of GLVQ [14] is to minimize

$$E = \sum_{i=1}^{m} f(g(\mathbf{x}^i)) \qquad (2)$$

via stochastic gradient descent. The update rules for GLVQ and many other LVQ algorithms can be derived using the above notation. In the following, the LVQ1 [9], RLVQ [1], GLVQ [14] and GRLVQ [6] algorithms will be reviewed, before introducing the proposed relevance determination methods.

2.1. LVQ1

Given a training sample $(\mathbf{x}, y) \in X$, by letting $f(x) = x$ and $g(\mathbf{x}) = \eta d_j$ where $d_j = d^2(\mathbf{x}, \mathbf{w}^j)$ with $\mathbf{w}^j$ being the closest prototype to $\mathbf{x}$ and $\{\lambda_i = 1\}_{i=1}^{m}$ (i.e. equal weights for regular Euclidean distance), with $\eta = 1$ if $\mathbf{x}$ is classified correctly (i.e. $c^j = y$) and $\eta = -1$ if $\mathbf{x}$ is classified incorrectly (i.e. $c^j \neq y$), the following stochastic gradient descent update rule may be derived for LVQ1 [9]:

$$\mathbf{w}_{t+1}^{j} = \begin{cases} \mathbf{w}_t^j + \alpha(\mathbf{x} - \mathbf{w}_t^j), & \text{if } c^j = y \\ \mathbf{w}_t^j - \alpha(\mathbf{x} - \mathbf{w}_t^j), & \textit{otherwise}, \end{cases} \qquad (3)$$

where α is the learning rate and the t subscripts denote prototype states at different training steps. However, it should be noted that the error function as defined here is highly discontinuous, and thus can lead to instabilities in the algorithm. GLVQ, discussed next, was designed to resolve this issue.

2.2. GLVQ

Here, $d_j = d^2(\mathbf{x}, \mathbf{w}^j)$ is defined where $\mathbf{w}^j$ is the closest prototype to $\mathbf{x}$ with label $c^j = y$ and $d_k = d^2(\mathbf{x}, \mathbf{w}^k)$ where $\mathbf{w}^k$ is the closest prototype to $\mathbf{x}$ with some other label. By letting

$$g(\mathbf{x}) = \frac{d_j - d_k}{d_j + d_k} \tag{4}$$

and

$$f_t(g(\mathbf{x})) = \frac{1}{1 + \exp^{-g(\mathbf{x})t}}, \tag{5}$$

which is a sigmoidal function that redefines the error function (Eq. 2) such that it is continuous over borders between the receptive fields for $\mathbf{w}^j$ and $\mathbf{w}^k$. When minimized, the error function yields the following update rules for $\mathbf{w}^j$ and $\mathbf{w}^k$ [14]:

$$\mathbf{w}^j_{t+1} := \mathbf{w}^j_t + \alpha\nu \frac{d_k}{(d_j + d_k)^2}(\mathbf{x} - \mathbf{w}^j_t) \tag{6}$$

$$\mathbf{w}^k_{t+1} := \mathbf{w}^k_t + \alpha\nu \frac{d_j}{(d_j + d_k)^2}(\mathbf{x} - \mathbf{w}^k_t) \tag{7}$$

where

$$\nu = f'_t(g(\mathbf{x})) = f_t(g(\mathbf{x}))(1 - f_t(g(\mathbf{x})). \tag{8}$$

GLVQ, unlike LVQ1 or the rest of Kohonen's original LVQ formulations, has been shown to be convergent [14, 6], although it is sensitive to the initialization of the prototype vectors. This is demonstrated in the experimental results of Section 4.

2.3. RLVQ and GRLVQ

The LVQ prototype update equations can be accompanied by updates that also alter the λ_i in Eq. (1) dynamically during training, hence allowing for an adaptive Euclidean metric. In RLVQ [1], LVQ1 training is adjusted such that the following weighting factor update rule is applied alongside Eq. (3):

$$\lambda_l := \begin{cases} \lambda_l - \beta(x_l - w^j_l)^2 & \text{if } c^j = y \\ \lambda_l + \beta(x_l - w^j_l)^2 & \textit{otherwise}, \end{cases} \tag{9}$$

for each l-th dimension where $\beta \in (0, 1)$ is a learning rate for the weighting factor adjustments. The weights are normalized at each update such that $\sum_{i=1}^{n} \lambda_i = 1$. The motivation for the above comes from Hebbian learning, the idea being that when $\mathbf{w}^j$ classifies the sample $\mathbf{x}$ correctly, the weights for the dimensions that contributed to the classification the most are increased, whereas the weights of those that contributed the least are decreased. When $\mathbf{w}^j$ incorrectly classifies $\mathbf{x}$, the weights for dimensions that contributed most are decreased, whereas the weights for dimensions that contributed the least are increased. GRLVQ [6] is an application of the above idea to GLVQ, such that the updates for the weights for the metric also follow a stochastic gradient descent on the error function defined by GLVQ.

One disadvantage of both RLVQ and GRLVQ is that they require the specification of an additional learning rate, β, which can be difficult to specify appropriately with respect to its α counterpart in the prototype updates. Another disadvantage is that they fail to take into consideration the additional statistical information provided by the remaining prototypes other than the ones currently being updated at a given training step when making relevance estimates. These issues are addressed with the following two proposed LVQ relevance determination algorithms.

3. Proposed Algorithms

The Fisher criterion, while ordinarily associated with Fisher's discriminant analysis [4], can also serve as an effective means for relevance determination when applied across individual data dimensions. Letting $\overline{x}^A = \frac{1}{N}\sum_{x^i \in A} x^i$ be the mean of a set of points A with cardinality N, the Fisher criterion score for a given individual dimension l is defined as

$$F(l) = \frac{S_B(l)}{S_W(l)}, \tag{10}$$

where

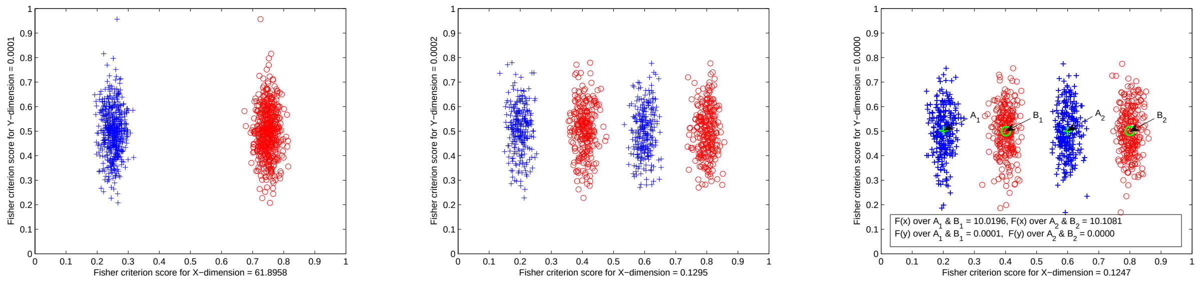
$$S_B(l) = \sum_{c=1}^{C} N^c \left(\overline{x}^{X^c}_l - \overline{x}^X_l\right)^2 \tag{11}$$

is the between-class variance and

$$S_W(l) = \sum_{c=1}^{C} \sum_{\mathbf{x} \in X^c} \left(x_l - \overline{x}^{X^c}_l\right)^2 \tag{12}$$

is the within-class variance over the l-th dimension.

With regard to relevance determination for LVQ, $F(l)$ could be calculated for each dimension over the entire training set X in advance of LVQ training and applied to the weighting factors in Eq. (1) by setting $\lambda_l = F(l)$ for all l to form a weighted metric. However, for many applications it is more desirable to have an online feature relevance training mechanism that is not reliant on having access to the entire training set at once. Two such online algorithms where estimation of the Fisher criterion score is integrated into the training scheme are presented next.



(a) (b) (c)

Figure 1. A simple 2D, 2-class example of how the Fisher criterion score (see Eq. (10)) can fail as a feature relevance metric over multi-modal distributions. (a) shows uni-modal class data distributions, linearly separable in the x-dimension, but with large overlap in the y-dimension. The score reflects the relevance of each dimension to class discrimination. (b) by comparison, shows the same number of data points, but with a multi-modal distribution (yet still linearly separable in x). The score is significantly lower for the x-dimension in this case. (c) shows the improvement provided by calculating the score between pairs of clusters with centers at points A_1, B_1, A_2 and B_2. See Section 3 for more details.

3.1. Algorithm 1

With the first algorithm, rather than calculating $F(l)$ over the data in X, at a given timestep t the score is estimated over the values of the prototype vectors in W. This is plausible since the distribution of the prototype vectors should approximate the distribution of the data over time. During training, certain prototypes will quantize more significant modes of the distribution than others, thus to account for this, weighted means and variances are calculated for each class based on the classification accuracy of each of the prototypes of that class, then the Fisher criterion score is calculated over the weighted means and variances for all classes. Firstly, the definition of $\mathcal{W}$ is altered to $\mathcal{W} = \{(\mathbf{w}^i, c^i, p^i) \subset \mathbb{R}^n \times \{1, \ldots, C\} \times \mathbb{R} \mid i = 1, \ldots, M\ \}$ where, given random variable $(\mathbf{x}, y)$, $p^i = p(\mathbf{x} \in R^i | y = c^i)$ is the conditional probability of x lying in receptive field R^i of prototype $\mathbf{w}^i$ given that $\mathbf{w}^i$ correctly classifies x. The p^i form probability distributions over class prototypes such that $\sum_{p^i \in \mathcal{W}^c} p^i = 1$ for each class c. A definition of the estimated Fisher criterion score may now be formed as

$$F(l) \simeq \hat{F}(l) = \frac{\hat{S}_B(l)}{\hat{S}_W(l)}, \tag{13}$$

where

$$S_B(l) \simeq \hat{S}_B(l) = \sum_{c=1}^{C} \frac{N^c}{N} \left(\hat{w}_l^{\mathcal{W}^c} - \hat{w}_l^{\mathcal{W}}\right)^2 \tag{14}$$

is the estimated between-class variance over the l-th dimension,

$$S_W(l) \simeq \tag{15}$$

$$\hat{S}_W(l) = \sum_{c=1}^{C} \frac{N^c}{N} \sum_{(\mathbf{w}^i, c^i, p^i) \in \mathcal{W}^c} p^i \left(w_l - \hat{w}_l^{\mathcal{W}^c}\right)^2 \tag{16}$$

is the estimated within-class variance over the l-th dimension, and

$$\hat{w}_l^{\mathcal{W}^c} = \sum_{(\mathbf{w}^i, c^i, p^i) \in \mathcal{W}^c} p^i w_l^i \tag{17}$$

is a weighted mean over the l-th dimension of prototypes in a given set $\mathcal{W}^c \subseteq \mathcal{W}$.

The λ_m relevance factors may then be updated at each timestep by taking a running mean of the normalized estimated Fisher criterion score:

$$\lambda_{l,t+1} := \lambda_{l,t} + \frac{\frac{\hat{F}(l)}{\sum_{l=1}^{n} \hat{F}(l)} - \lambda_{l,t}}{t+1}. \tag{18}$$

While the Fisher criterion score is suitable for feature relevance determination in many cases, its main drawback is that it does not cope well with multi-modal feature distributions. An example of this is shown in Figure 1. This problem remains in the estimation proposed above, since Eq. (14) and Eq. (16) are calculated over all class prototypes. The second proposed algorithm was designed to account for this.

3.2. Algorithm 2

The second proposed algorithm is based on the idea of calculating the Fisher criterion score between single prototype vectors of opposing classes, where the assumption is made that each class prototype vector may be quantizing different modes of the underlying class distribution. During training, Gaussian kernels are used to maintain estimates of the accuracies of each of the prototypes over the parts the data distribution accounted for by each of their receptive fields. At a given training step, the nearest single prototypes of each class to the training sample are found, and their Gaussian kernels are used to calculate an estimate of the Fisher criterion score for that local portion of the distribution, which is subsequently averaged over the entire training period.

The definition of $\mathcal{W}$ is this time altered to accommodate a Gaussian estimate of the accurate portion of the receptive field for each prototype, such that

$$\begin{aligned}\mathcal{W} = \{ \ \big(\mathbf{w}^i, c^i, \mathcal{N}(\mathbf{x};\mu^i,\Sigma^i)\big) \subset \mathbb{R}^n \times \{1,\ldots,C\} \times \\ (\mathbb{R}^n \times \mathbb{R}^{n\times n}) \mid i = 1,\ldots,M \ \}, \qquad (19)\end{aligned}$$

where $\mathcal{N}$ approximates $\tilde{R}^i = \big\{\mathbf{x} \in R^i | y = c\big\}$ with mean μ^i and covariance matrix $\Sigma^i = \mathrm{diag}([s_1^i,\ldots,s_n^i])$ where the $\big\{s_l^i\big\}_{l=1}^n$ are variances along each l-th dimension. During LVQ training, given a random sample $(\mathbf{x}, y) \in X$ at training step t, if the closest prototype $\mathbf{w}^j$ classifies x correctly, i.e. $c^j = y$, then μ_l^j and s_l^j are updated in each l-th dimension as follows [8]:

$$\mu_{l,t}^j := \mu_{l,t-1}^j + \frac{x_l - \mu_{l,t-1}^j}{t} \qquad (20)$$

$$\hat{s}_{l,t}^j := \hat{s}_{l,t-1}^j + (x_l - \mu_{l,t-1}^j)(x_l - \mu_{l,t}^j) \qquad (21)$$

where $\mu_{l,t}^j$ is the running mean estimate and $s_{l,t}^j = \frac{\hat{s}_{l,t}^j}{t-1}$ is the running variance estimate for the l-th dimension at training step t. If $c^j \neq y$, then the above updates are not performed. Assuming a sufficient number of updates have been performed on the relevant prototypes up until step t, a Fisher criterion score estimate may be calculated between

$$\begin{aligned}\mathcal{W}' = \{\omega^k = \Big(\mathbf{w}^k, c^k, \mathcal{N}(\mathbf{x};\mu^k,\Sigma^k)\Big) \in \mathcal{W} \\ \mid \forall \mathbf{w}^i, c^i = c^k, d(\mathbf{x},\mathbf{w}^k) \leq d(\mathbf{x},\mathbf{w}^i) \ \}, \\ (22)\end{aligned}$$

the closest prototypes of different classes (including $\mathbf{w}^j$), as follows:

$$F(l) \simeq \tilde{F}(l) = \frac{\tilde{S}_B(l)}{\tilde{S}_W(l)}, \qquad (23)$$

where

$$S_B(l) \simeq \tilde{S}_B(l) = \frac{1}{C}\sum_{c=1}^{C}(\mu_l^c - \overline{\mu}_l)^2 \qquad (24)$$

is the between-class variance estimate in the l-th dimension with

$$\overline{\mu}_l = \frac{1}{C}\sum_{\omega^k \in \mathcal{W}'} \mu_l^k, \qquad (25)$$

and

$$S_W(l) \simeq \tilde{S}_W(l) = \sum_{\omega^k \in \mathcal{W}'} s_l^k \qquad (26)$$

is the within-class variance estimate in the l-th dimension. The relevance factors may then be updated in a similar way to Eq. (18), this time using the new estimates:

$$\lambda_{l,t+1} := \lambda_{l,t} + \frac{\frac{\tilde{F}(l)}{\sum_{l=1}^{n}\tilde{F}(l)} - \lambda_{l,t}}{t+1}. \qquad (27)$$

Since each prototype carries an accompanying Gaussian kernel that estimates its accuracy, it is now possible to estimate the Fisher criterion score using only single prototypes from each class, as opposed to the previous algorithm where multiple prototypes in each class have to be considered to achieve variance estimates. Though the model is made more complex, it is more capable of successfully handling the multimodal distribution issue described in Fig. 1 as shown by the experimental results in the next section.

4. Experiments

The proposed algorithms were evaluated over simulated data, datasets from the UCI repository, and a real-world dataset from a cognitive robotics object affordance learning experiment. In the following, the datasets are described in more detail and experimental results are provided in Section 4.1. Two simulated datasets were proposed in [1, 6], the first of which was replicated for the experiments here. The data is composed of three classes, each separated into two clusters with some small overlap to form multi-modal class data distributions in the first two dimensions. Eight further dimensions are generated from the first two dimensions as follows: assuming (x_1, x_2) is one data point, $x_3 = x_1 + \eta_1, \ldots, x_6 = x_1 + \eta_4$ is chosen where η_i comprises normally-distributed noise with variances $0.05, 0.1, 0.2$, and 0.5 respectively. The remaining $x_7, \ldots, x_{10}$ components contain pure noise uniformly distributed in

$[-0.5, 0.5]$ and $[-0.2, 0.2]$. This dataset is multi-modal for each class in the two relevant dimensions and thus provides a good test for the potential difference between the two proposed algorithms.

Dataset	# Features	# Samples	# Classes
Simulated	10	90	3
Iris	4	150	3
Ionosphere	34	351	2
Wine	13	178	3
Soybean	35	47	4
WBC	30	569	2
Affordance	11	160	2

Table 1. An attribute list for the datasets in Section 4.

Five different datasets from the UCI repository [5] were tested: Fisher's Iris dataset, the ionosphere dataset, the wine dataset, the soybean dataset (small), and the Wisconsin breast cancer (WBC) dataset. A dataset from a cognitive robotics object affordance learning experiment [13] was also tested. It consists of eight household objects separated into two classes, four rolling objects and four non-rolling objects, and labeled as such, accompanied by eleven different shape features, two of which measure the curvature of 3D points from stereo images of the objects and the remainder of which were derived from 2D silhouettes of the objects.

4.1. Results

The primary goal of the investigation was to evaluate whether or not the new algorithms when applied to standard LVQ methods such as LVQ1 and GLVQ offer performance improvements over those methods in their original form, as well as over other relevance determination techniques for LVQ, such as RLVQ and GRLVQ. The results of these comparisons are outlined in Table 2 and are discussed in more detail in the following. In the results, the proposed Fisher criterion score-based relevance determination algorithms are referred to as FC1LVQ1 and FC2LVQ1 respectively when applied to LVQ1, and FC1GLVQ and FC2GLVQ when applied to GLVQ.

A secondary consideration was to test the methods under the duress of various different conditions. GLVQ, for example is known to perform poorly if the prototype vectors are not initialized within the data distribution [12], thus in our evaluations, both random prototype initializations as well as initializations where the prototypes are placed at the mean points of class clusters were considered. Note that random prototype initialization in this case refers to selecting random values for each prototype dimension scaled within the data range. K-means clustering was used to determine class clusters in the latter case.

The performance of LVQ algorithms over short training periods is not often considered in the literature, which tends to favour evaluations of the algorithms over several hundred training epochs until convergence is reached. Given that LVQ algorithms have online training mechanisms, and that the relevance determination techniques proposed above were explicitly developed to also function online, sample-by-sample without access to the rest of the training set, such short-term training evaluations are important if the methods are to be considered useful in real-world online settings, e.g. cognitive robotics [13], where the entire training set is often unavailable at any given point during training.

Thus, the results in Table 2 are divided into four main evaluations: both 1 epoch and 300 epochs of training from random initialization, and both 1 epoch and 300 epochs of training from class cluster mean initialization. The 300 epoch sessions used the relatively slow learning rates of $\alpha = 0.1$ for the prototype updates (cf. Eq. (3), Eq. (6) & Eq. (7)) and $\beta = 0.01$ for the dimensional relevance updates where required (cf. Eq. (9)), whereas the 1 epoch training sessions used the faster rates of $\alpha = 0.3$ and $\beta = 0.1$. Note that the FC1 and FC2 methods do not require the additional β learning rate. In each of the 1 epoch evaluations, 20 trials of ten-fold cross validation were performed with random data orderings in each trial, and results were averaged over test data performance, whereas in the 300 epoch evaluations, 5 trials were performed. 10 prototypes were used for every dataset and the data dimensions were scaled prior to training.

The results in Table 2 show that when trained over a single epoch from random initialization, of the algorithms tested FC2LVQ1 and FC2GLVQ achieved higher mean classification scores than their counterparts in many cases. Over long-term training of 300 epochs from random initialization, the results for all algorithms aside from GLVQ, tend to improve with FC2LVQ1 and FC2GLVQ again tending to be competitive with their counterparts. It is worth noting here the impact relevance determination has on improving the results of GLVQ when exposed to poor prototype initialization. When the prototypes are initialized optimally at the class cluster mean points the results tend to improve dramatically across all of the

Dataset	LVQ1	RLVQ1	**FC1LVQ1**	**FC2LVQ1**	GLVQ	GRLVQ	**FC1GLVQ**	**FC2GLVQ**
Random Initialization, 1 Epoch of Training, 20 Trials								
Sim	53± 18%	64± 22%	54± 19%	**69± 18%**	37± 17%	63± 22%	51± 20%	**70± 19%**
Iris	90± 8%	91± 9%	93± 9%	**95± 5%**	63± 24%	**89± 13%**	83± 19%	88± 15%
Iono	81± 8%	75± 11%	**85± 6%**	84± 7%	66± 13%	80± 9%	82± 7%	**84± 7%**
Wine	93± 6%	79± 13%	92± 9%	**94± 6%**	52± 19%	92± 8%	85± 14%	**94± 7%**
Soy	**89± 17%**	83± 24%	89± 18%	85± 21%	34± 27%	84± 22%	83± 21%	**85± 20%**
WBC	92± 4%	86± 8%	93± 4%	**93± 3%**	71± 19%	93± 5%	90± 10%	**94± 3%**
Afford	97± 7%	93± 10%	98± 4%	**99± 3%**	78± 22%	96± 9%	84± 20%	**98± 6%**
Random Initialization, 300 Epochs of Training, 5 Trials								
Sim	79± 14%	79± 13%	77± 16%	**87± 12%**	38± 17%	**96± 7%**	90± 12%	94± 9%
Iris	92± 7%	92± 8%	95± 5%	**96± 5%**	47± 24%	96± 5%	91± 16%	**96± 4%**
Iono	85± 7%	80± 10%	**86± 8%**	85± 7%	60± 16%	**90± 5%**	90± 6%	89± 6%
Wine	95± 5%	77± 11%	95± 5%	**96± 5%**	42± 18%	96± 5%	97± 4%	**98± 3%**
Soy	99± 6%	97± 10%	**100± 4%**	98± 7%	33± 26%	97± 8%	**97± 7%**	96± 9%
WBC	93± 3%	87± 7%	**94± 3%**	**94± 3%**	62± 20%	96± 3%	96± 3%	**96± 2%**
Afford	**99± 2%**	95± 7%	**99± 2%**	99± 3%	67± 24%	**99± 2%**	**99± 2%**	**99± 2%**
Class Cluster Mean Initialization, 1 Epoch of Training, 20 Trials								
Sim	82± 12%	**98± 5%**	78± 17%	93± 8%	90± 9%	91± 9%	85± 13%	**93± 8%**
Iris	**96± 5%**	**96± 5%**	**96± 5%**	**96± 5%**	95± 5%	95± 5%	95± 5%	**96± 5%**
Iono	87± 6%	80± 10%	**88± 6%**	**88± 6%**	**90± 5%**	88± 6%	89± 5%	**90± 5%**
Wine	95± 5%	86± 11%	**96± 5%**	**96± 5%**	**97± 4%**	97± 5%	97± 5%	97± 5%
Soy	**100± 2%**	95± 10%	100± 3%	99± 5%	**100± 2%**	99± 4%	**100± 2%**	99± 5%
WBC	**95± 3%**	88± 7%	94± 3%	94± 3%	96± 3%	96± 3%	**97± 3%**	95± 3%
Afford	**99± 2%**	98± 4%	**99± 2%**	**99± 2%**	**99± 2%**	**99± 2%**	**99± 2%**	**99± 2%**
Class Cluster Mean Initialization, 300 Epochs of Training, 5 Trials								
Sim	84± 11%	86± 16%	87± 12%	**91± 10%**	90± 9%	**97± 6%**	90± 10%	96± 8%
Iris	96± 5%	95± 6%	**96± 4%**	96± 5%	96± 6%	95± 5%	**97± 4%**	96± 4%
Iono	88± 5%	82± 9%	**89± 5%**	88± 5%	89± 5%	90± 5%	90± 5%	**91± 5%**
Wine	96± 5%	82± 12%	**97± 4%**	96± 5%	97± 4%	**98± 3%**	**98± 3%**	**98± 3%**
Soy	**100± 0%**	94± 11%	99± 6%	98± 7%	**100± 0%**	98± 8%	99± 5%	99± 6%
WBC	**96± 2%**	89± 5%	95± 3%	95± 3%	96± 3%	96± 3%	**97± 2%**	**97± 2%**
Afford	**99± 2%**	98± 3%	**99± 2%**	**99± 2%**	99± 3%	**99± 2%**	**99± 2%**	**99± 2%**

Table 2. 10-Fold cross validation, 10 prototypes. Highest scores for LVQ1 & GLVQ based algorithms are shown in bold.

classifiers in short-term training, with both FC1 and FC2 relevance determination doing well over both short-term and long-term training periods, with FC1 out-performing FC2 in some cases and vice versa. Over all the evaluations, FC1GLVQ and FC2GLVQ trained over 300 epochs with class cluster mean initialization tended to score well when compared with the other methods. It should also be noted that, when the class distribution in the data is multi-modal, as is the case with the simulated dataset, FC2-based methods tend to be a better choice than FC1-based methods, as predicted.

A third consideration was to compare the new methods to a state-of-the-art batch method such as the *support vector machine (SVM)*. Batch methods, as opposed to online methods that are trained sample-by-sample, have access to the entire training set during training, and therefore usually provide superior results. Table 3 shows the results of a comparison between FC1GLVQ, FC2GLVQ and a multi-class SVM trained with a radial basis function (RBF) kernel [2]. For this comparison, the results for FC1GLVQ and FC2GLVQ from the 300 epoch, class cluster mean-initialized evaluation described previously were used, while ten-fold cross validation over five trials was also used for the SVM, where the test data results were averaged over the five trials and SVM parameters were optimized using cross validation over the training data prior to training. The results show both FC1GLVQ and FC2GLVQ performing well when compared with SVM over the various datasets, particularly in the case of the simulated multi-modal dataset.

It is difficult to evaluate the performance of the algorithms with respect to the estimation of the λ_l weighting factors themselves, but examples of the

Dataset	FC1GLVQ	FC2GLVQ	SVM
Simulated	90±10%	**96±8%**	78±14%
Iris	**97±4%**	96±4%	96±6%
Ionosphere	90±5%	91±5%	**94±4%**
Wine	**98±3%**	**98±3%**	**98±3%**
Soybean	99±5%	99±6%	**100±0%**
WBC	97±2%	97±2%	**98±2%**
Affordance	**99±2%**	**99±2%**	99±3%

Table 3. FC1GLVQ & FC2GLVQ versus SVM. Highest mean scores are shown in bold.

mean values for certain datasets are provided here. For the simulated dataset, $\lambda_{\mathrm{FC1GLVQ}} = \{0.10, 0.42, 0.07, 0.06, 0.10, 0.06, 0.04, 0.03, 0.07, 0.04\}$ and $\lambda_{\mathrm{FC2GLVQ}} = \{0.40, 0.43, 0.06, 0.01, 0.01, 0, 0, 0, 0, 0\}$, thus demonstrating that FC2GLVQ does indeed do a better job of handling the multi-modal distribution. For the Iris dataset, $\lambda_{\mathrm{FC1GLVQ}} = \{0.02, 0.02, 0.55, 0.40\}$ and $\lambda_{\mathrm{FC2GLVQ}} = \{0.03, 0.07, 0.37, 0.53\}$. For the object affordance dataset, $\lambda_{\mathrm{FC1GLVQ}} = \{0.04, 0.56, 0.05, 0.05, 0.03, 0.05, 0.04, 0.04, 0.01, 0.09, 0.05\}$ and $\lambda_{\mathrm{FC2GLVQ}} = \{0.05, 0.34, 0.07, 0.07, 0.07, 0.08, 0.01, 0.08, 0.06, 0.12, 0.06\}$, where one of the 3D curvature features is favoured in each case.

5. Conclusion

In conclusion, two new relevance determination algorithms have been proposed for LVQ that exploit the positioning of prototypes in the input feature space to calculate Fisher criterion score estimates in the input dimensions for an adaptive metric. An advantage provided by these methods over other metric-adaptive LVQ methods based on gradient descent, is that they do not require a learning rate or other parameters to be specified. Moreover, they provide incremental update rules that operate alongside regular LVQ update rules and can therefore be applied to any algorithms based on the general LVQ paradigm. Experimental evaluations were provided under various stress conditions and over various datasets and the proposed methods were shown to perform competitively against various other LVQ-based methods, and against SVM. With regard to future work, it would be interesting to apply the proposed techniques to prototype-based methods other than LVQ, such as supervised neural gases.

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